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FORECASTING NIGERIAN EQUITY VOLATILITY: A COMPARATIVE ASSESSMENT OF GARCH, EGARCH, AND STOCHASTIC VOLATILITY MODELS

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ABSTRACT

This study conducts a comprehensive comparative assessment of volatility forecasting models for the Nigerian equity market, evaluating the performance of symmetric GARCH, asymmetric GARCH-family models, and a latent Stochastic Volatility (SV) model a class of models previously absent from empirical applications in Nigeria. Utilizing monthly data from 2010 to 2023, we employ a two-stage empirical framework: first estimating in-sample characteristics using Quasi-Maximum Likelihood for GARCH models and Bayesian Markov Chain Monte Carlo (MCMC) for the SV model, and then rigorously testing out-of-sample forecast accuracy through a rolling-window scheme. Forecasts are evaluated against a superior realized volatility proxy using robust loss functions and the Model Confidence Set (MCS) procedure. Our findings reveal that the Stochastic Volatility model demonstrates statistically superior forecasting performance, achieving the lowest Mean Squared Error (4.125×10^{-5}), Mean Absolute Error (1.521×10^{-3}), and QLIKE loss, and is the sole model retained in the MCS at a 75% confidence level. The asymmetric GJR-GARCH model ranked second, followed by EGARCH, with symmetric GARCH(1,1) performing worst. The SV model's superiority is particularly pronounced during high-volatility regimes, indicating its unique capacity to capture independent "volatility shocks" characteristic of frontier markets. These results suggest that Nigerian equity volatility is more faithfully represented by a latent stochastic process than by deterministic GARCH specifications, necessitating a paradigm shift in risk modeling practices for the NGX and similar frontier markets.

Keywords: Volatility Forecasting; Stochastic Volatility; GARCH Models; Model Confidence Set; Frontier Markets; Risk Management.

1. INTRODUCTION

The intrinsic volatility of financial asset returns represents the cornerstone of modern finance, underpinning critical domains such as asset pricing, portfolio optimization, Value-at-Risk (VaR) forecasting, and the pricing of derivative instruments. Accurate estimation and forecasting of this latent volatility are not merely academic exercises but are essential for informed financial decision-making by institutional investors, risk managers, and regulatory authorities. In emerging and frontier markets, where financial systems are often characterized by pronounced instability, lower liquidity, informational inefficiencies, and heightened susceptibility to both domestic policy shocks and global financial contagion, the ability to robustly model and predict market volatility acquires an even greater, practical significance. The Nigerian equity market, epitomized by the Nigerian Exchange Group (NGX), stands as the second-largest bourse in Africa by market capitalization and serves as the primary barometer for the continent's largest economy. Historically, this market has exhibited pronounced cycles of boom and bust, with periods of frenetic growth such as the pre-2008 banking-

led bull run often punctuated by severe, protracted corrections, as witnessed during the global financial crisis, the 2014-2016 oil price collapse, and the COVID-19 pandemic shock of 2020 (Oduyemi et al., 2022; SEC Nigeria, 2023). These episodes underscore the market's inherent susceptibility to domestic macroeconomic instabilities notably exchange rate volatility, inflationary pressures, and fiscal and monetary policy shifts as well as its vulnerability to volatile global commodity prices and capital flow reversals. Consequently, developing and identifying the most robust, empirically validated models to capture and forecast the dynamic behavior of Nigerian stock market volatility is a pressing research imperative with substantial implications for enhancing market resilience, designing effective hedging strategies, strengthening financial stability frameworks, and improving investment outcomes.

The theoretical and empirical evolution of volatility modeling has been one of the most prolific areas in financial econometrics since the seminal contribution of Engle (1982) and Bollerslev (1986), who introduced the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH) models. These foundational frameworks revolutionized the field by explicitly parameterizing the time-varying nature of return variance, thereby formally capturing the well-documented stylized fact of volatility clustering where large price changes tend to be followed by large changes, and small changes by small changes. The standard GARCH(1,1) model, with its parsimonious structure and symmetric response to positive and negative shocks, provides a powerful and widely adopted baseline. However, a vast body of subsequent empirical evidence, particularly from developed equity markets, has firmly established the prevalence of the asymmetric “leverage effect” (Black, 1976) or “volatility feedback effect” (Bekaert & Wu, 2000), wherein negative return shocks ("bad news") tend to increase future volatility more than positive shocks ("good news") of equal magnitude.

This critical stylized fact necessitated the development of more sophisticated, asymmetric volatility models. Nelson’s (1991) Exponential GARCH (EGARCH) model, which models the log of the conditional variance, and Glosten, Jagannathan, and Runkle’s (1993) GJR-GARCH model, which incorporates a dummy variable for negative innovations, directly accommodate this asymmetry. These models have become standard tools in the financial econometrician's toolkit for capturing the nuanced dynamics of financial volatility where downside risk is priced differently.

Parallel to the evolution of the GARCH-family, an alternative and theoretically distinct paradigm emerged in the form of Stochastic Volatility (SV) models, championed by Taylor (1982, 1994). Unlike GARCH-class models, where the conditional variance for the next period is a deterministic function of past squared returns and past variances, SV models treat volatility as an unobserved, latent stochastic process driven by its own separate innovation term. This fundamental architectural difference offers several theoretical and empirical advantages. First, it provides a more flexible and realistic representation of the information flow into volatility, acknowledging that volatility shocks are not perfectly correlated with return shocks—a feature often observed in high-frequency data (Shephard, 2005).

Second, the continuous-time limit of discrete-time SV models aligns naturally with the seminal stochastic volatility models used in options pricing theory, such as the Heston (1993) model, providing a stronger link between time-series econometrics and derivative asset pricing. Third, the SV framework can more readily accommodate and model long memory properties in volatility, which are challenging to capture parsimoniously within the standard GARCH framework. The estimation of SV models, historically constrained by significant computational complexity due to the high-dimensional integration of latent variables, has been dramatically facilitated over the last two decades by the advent and refinement of simulation-based Bayesian inference techniques, particularly Markov Chain Monte Carlo (MCMC) methods like the Gibbs sampler (Jacquier, Polson, & Rossi, 1994; Kastner, 2016). These advances have rendered sophisticated SV models increasingly accessible for empirical application and comparative forecasting exercises.

Despite this rich and mature global literature, the comparative forecasting performance of these competing volatility modeling paradigms symmetric GARCH, asymmetric GARCH-family, and latent stochastic volatility models in the specific context of frontier African markets like Nigeria remains notably underexplored and inconclusive. Existing scholarly work on the NGX has predominantly, and often exclusively, focused on applying and comparing various specifications within the GARCH family. For instance, studies by Olowe (2009), Ezeamama and Ajudua (2016),

and more recently, Adesina (2021) and Okafor and Ujah (2023) have applied GARCH, EGARCH, IGARCH, and FIGARCH models to Nigerian stock index returns, generally confirming the presence of high volatility persistence and, in several cases, evidence of asymmetry. However, this extant literature exhibits two critical and interrelated gaps. First, there is a striking lack of comprehensive, head-to-head forecasting comparisons that rigorously evaluate the out-of-sample predictive accuracy of symmetric (GARCH), asymmetric (EGARCH, GJR-GARCH), and the latent stochastic volatility (SV) model using a consistent, robust, and statistically rigorous evaluation framework. Most studies either focus exclusively on in-sample fit or conduct comparisons only within the confines of the GARCH family, thereby ignoring a whole class of models that may be theoretically superior. Second, the choice of a superior volatility proxy for model validation is crucial, as forecast accuracy is contingent on the benchmark. Many studies on emerging markets rely on squared daily returns as a noisy and inefficient proxy for latent volatility, a practice known to distort forecast evaluation and model ranking (Andersen & Bollerslev, 1998; Patton, 2011). The use of a more efficient ex-post volatility measure, such as realized volatility computed from intraday high-frequency data, remains exceptionally rare in the Nigerian and broader African context due to data constraints. However, even lower-frequency approximations, such as the Parkinson (1980) range-based estimator or realized volatility computed from carefully sampled intraday data (when available), can provide a significant improvement in measurement accuracy over simple squared returns, leading to more reliable model comparisons.

This study, therefore, seeks to make a substantive contribution by directly addressing these identified gaps. Its primary objective is to conduct a rigorous, methodologically sound comparative assessment of the forecasting performance of the leading volatility models for the Nigerian equity market. Specifically, the research is designed to: (i) estimate and compare the in-sample statistical characteristics and fitted dynamics of a standard GARCH(1,1), an EGARCH(1,1), a GJR-GARCH(1,1), and a Bayesian Stochastic Volatility (SV) model for the NGX All-Share Index total returns; (ii) evaluate their out-of-sample volatility forecasting accuracy over multiple practical horizons (one-day, one-week, one-month) using an expanding or rolling-window scheme to ensure robustness; and (iii) validate the forecasts against a superior proxy for realized volatility, employing robust loss functions and the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011) to statistically discern superior predictive ability. The central research question guiding this empirical inquiry is: Among the classes of symmetric GARCH, asymmetric GARCH-family, and latent stochastic volatility models, which specification provides the most accurate and statistically reliable forecasts of equity market volatility in Nigeria, and does this superiority hold across different forecasting horizons and market regimes (tranquil vs. crisis periods)?

The significance of this study is threefold. Methodologically, it represents one of the first comprehensive applications of a Bayesian MCMC-estimated Stochastic Volatility model to Nigerian financial data, benchmarked against the full suite of established GARCH-family models. It advances the empirical toolkit for frontier market analysis by implementing a sophisticated forecasting evaluation protocol that emphasizes out-of-sample testing and employs an enhanced volatility proxy, moving the literature beyond in-sample diagnostics. Theoretically, the findings will shed critical light on the underlying data-generating process of Nigerian equity returns.

A dominant performance by asymmetric GARCH models would provide strong, forecast-based evidence of the leverage effect in a frontier market context, informing asset pricing models. Conversely, if the SV model demonstrates superior forecasting power, it would suggest that Nigerian equity volatility evolves with a significant independent stochastic component, a finding with profound implications for the specification of risk models and the potential pricing of volatility derivatives locally. Practically, the identification of a dominant forecasting model has direct, actionable utility for a wide range of stakeholders. For institutional investors, asset managers, and proprietary trading firms, it provides empirical guidance for selecting the most reliable risk models for dynamic portfolio optimization, VaR calculation, and the hedging of equity exposures. For the NGX and the Securities and Exchange Commission (SEC), a deeper understanding of volatility dynamics and its predictability can inform the calibration of market-wide circuit breakers, margin requirements, and broader financial stability surveillance. For multinational corporations with Nigerian operations,

improved volatility forecasts can enhance capital budgeting, corporate treasury, and enterprise risk management decisions.

2. LITERATURE REVIEW

2.1 Theoretical Foundations of Volatility Modeling

The modeling of financial market volatility has evolved through distinct theoretical paradigms, each addressing the empirical deficiencies of its predecessors. The foundational breakthrough came with Engle's (1982) Autoregressive Conditional Heteroskedasticity (ARCH) model, which formally conceptualized volatility as a conditional variance that changes systematically over time based on past squared innovations. Bollerslev's (1986) Generalized ARCH (GARCH) extension introduced autoregressive persistence in the conditional variance itself, yielding the parsimonious and highly influential GARCH(1,1) model. This framework elegantly captured the stylized fact of volatility clustering, where periods of high turbulence are followed by high turbulence, and calm periods by calm periods. The theoretical appeal of GARCH models lies in their deterministic structure: conditional volatility for the next period is a known function of past observed returns and past conditional variances.

However, the empirical regularity of the leverage effect (Black, 1976) necessitated a theoretical shift. The observation that negative returns increase future volatility more than positive returns of equal magnitude challenged the symmetry assumption in standard GARCH. This led to the development of asymmetric volatility models, primarily motivated by two (non-mutually exclusive) economic theories: the financial leverage hypothesis and the volatility feedback hypothesis (Bekaert & Wu, 2000). The former posits that a drop in equity value increases a firm's debt-to-equity ratio (leverage), making the stock riskier and thus increasing its volatility. The latter suggests that an anticipated increase in future volatility raises the required rate of return, causing an immediate price drop. Nelson's (1991) Exponential GARCH (EGARCH) model addressed this by formulating the conditional variance in logarithmic terms, ensuring positivity without constraints and allowing for an asymmetric response function. Concurrently, Glosten, Jagannathan, and Runkle's (1993) GJR-GARCH model introduced a threshold term to differentiate the impact of positive and negative shocks. These models represent a significant theoretical advancement by formally incorporating the sign of the innovation into the volatility process.

A theoretically distinct paradigm emerged with the Stochastic Volatility (SV) model (Taylor, 1982, 1994). Its core theoretical premise is that volatility is an unobserved, latent variable that follows its own stochastic process, driven by a separate shock term that is correlated with, but not identical to, the return shock. This structure offers several theoretical advantages. First, it provides a more flexible and realistic mechanism for information arrival, where news can affect prices and volatility independently. Second, it aligns seamlessly with continuous-time models used in options pricing theory, such as the Heston (1993) model, creating a unifying framework for time-series econometrics and derivative pricing. Third, the SV model can naturally generate a higher degree of kurtosis (fat tails) in the unconditional return distribution than standard GARCH models (Shephard, 2005). The estimation challenge posed by its latent structure was theoretically overcome with the application of Bayesian inference and Markov Chain Monte Carlo (MCMC) methods, as demonstrated by Jacquier, Polson, and Rossi (1994).

2.2 Empirical Evidence from Global and Developed Markets

A vast body of empirical literature has tested and compared these theoretical models, primarily in developed markets. Early comparative studies, such as those by Heynen and Kat (1994) and Pagan and Schwert (1990), often found that GARCH-family models provided reasonable in-sample fits but had mixed out-of-sample forecasting success. A landmark study by Andersen and Bollerslev (1998) demonstrated that when evaluated against high-frequency realized volatility (a more efficient proxy), GARCH models could indeed provide accurate forecasts, resolving earlier skepticism.

The comparative performance between symmetric and asymmetric GARCH models is well-documented. For major indices like the S&P 500, the asymmetric effect is robust, leading to the consistent outperformance of EGARCH and GJR-GARCH over standard GARCH in both in-sample fit and out-of-sample forecasting, particularly during crisis periods (Chiang & Wang, 2011; Degiannakis & Xekalaki, 2020). The comparison between GARCH-family and SV models has yielded

more nuanced findings. Early work by Kim, Shephard, and Chib (1998) and Jacquier et al. (1994) found that SV models often provided a superior in-sample fit to return data. In forecasting, studies like that of Poon and Granger (2003) in their extensive survey noted that while no single model dominates universally, SV models often perform comparably or better, especially over longer horizons, due to their more flexible dynamic structure. More recent studies utilizing advanced estimation and high-frequency benchmarks, such as those by Chiriac and Voev (2011) and Bekierman and Manner (2022), often find that carefully specified SV models compete fiercely with the best asymmetric GARCH models, with results sensitive to the evaluation metric, forecast horizon, and volatility proxy.

2.3 Empirical Evidence from Emerging and Frontier Markets

The empirical landscape in emerging markets (EMs) and frontier markets (FMs) is more heterogeneous, reflecting diverse market microstructures, levels of integration, and institutional maturity. A common finding across EMs is the presence of extremely high volatility persistence and pronounced asymmetry, often more severe than in developed markets, due to greater informational frictions and sensitivity to global risk sentiment (Arouri, Lahiani, & Nguyen, 2011).

In the African context, studies have largely concentrated on applying GARCH-family models. For South Africa's JSE, research (e.g., Muteba Mwamba, 2012; Phiri, 2017) confirms strong asymmetry and leverage effects, with EGARCH often being favored. For other major African markets like Kenya (Ngunyi, Mutua, & Waweru, 2019) and Egypt (Elsayed & Sousa, 2021), similar conclusions have been drawn regarding the superiority of asymmetric GARCH specifications.

Focusing on Nigeria, the empirical literature has grown but remains predominantly focused on the GARCH universe. Foundational work by Olowe (2009) established the high persistence of NGX volatility using GARCH models. Subsequent studies have explored various extensions:

Asymmetry: Studies by Ezeamama and Ajudua (2016) and more comprehensively by Okafor and Ujah (2023) have confirmed the presence of a leverage effect in the NGX, with EGARCH and GJR-GARCH models showing better in-sample performance than standard GARCH.

Long Memory and Structural Breaks: Recognizing potential instability, Adesina (2021) applied a GARCH-MIDAS model to incorporate macroeconomic determinants, while others have used FIGARCH or models with structural break dummies (Iheanacho, 2020).

Forecasting Comparisons: Limited work exists on formal out-of-sample forecast competitions. Exceptions include studies like that of Ibe and Ozuomba (2021), which compared the forecasting accuracy of GARCH, EGARCH, and GJR-GARCH for sectoral indices, finding mixed results dependent on the sector. However, these studies almost universally use squared daily returns as the volatility proxy, a known source of evaluation noise.

2.4 Identification of Research Gaps

The preceding review reveals a critical and multi-layered gap in the literature, particularly concerning frontier markets like Nigeria:

i. Absence of Stochastic Volatility Models: There is a near-total absence of empirical applications of the Stochastic Volatility model to Nigerian equity data. The entire class of latent volatility models remains unexplored in comparative studies, creating a significant blind spot. It is empirically unknown whether the theoretical advantages of the SV framework translate into superior forecasting performance in the unique, often crisis-prone environment of the NGX.

ii. Insufficient Forecasting Competitions: Existing Nigerian studies are overwhelmingly focused on in-sample analysis. Where out-of-sample forecasts are compared, they are typically limited to contests within the GARCH family (GARCH vs. EGARCH). A comprehensive horse race that includes the SV model as a competitor, conducted over multiple forecast horizons and using a robust rolling-window design, is lacking.

iii. Deficient Volatility Measurement: The reliance on noisy proxies like squared returns for forecast evaluation is a major methodological weakness. This practice can lead to misleading model rankings (Patton, 2011). The literature on Nigeria has yet to adopt superior realized volatility measures—even range-based estimators (Parkinson, 1980) or realized volatility from available

intraday data—which are standard in contemporary international forecasting comparisons.

iv. Lack of Robust Statistical Inference: Model comparisons often rely on simple point estimates of loss functions (MSE, MAE) without formal statistical tests to determine if differences in forecast accuracy are significant. The application of modern procedures like the Model Confidence Set (MCS) (Hansen et al., 2011), which statistically identifies a set of superior models, is absent from the Nigerian volatility forecasting literature.

Therefore, this study contributes by directly addressing these four gaps. It will: (i) introduce and estimate a Bayesian MCMC Stochastic Volatility model for the NGX; (ii) conduct a rigorous out-of-sample forecasting competition against symmetric and asymmetric GARCH benchmarks across multiple horizons; (iii) employ a superior realized volatility proxy (using high-frequency intraday data or the best available approximation) for evaluation; and (iv) apply the MCS procedure to provide statistically rigorous conclusions on model superiority. By doing so, this research provides a definitive, methodologically sound comparison of volatility modeling paradigms for the Nigerian equity market.

3. METHODOLOGY

This study employs a comprehensive empirical framework to compare the forecasting performance of three major classes of volatility models for the Nigerian equity market. We begin by defining our dependent variable, realized volatility (RV), constructed as the sum of squared intraday returns over daily intervals $RVt = \sum i=1 Mrt,i2$, where rt,i denotes the i-th intraday return on day t and M is the number of intraday observations (Andersen & Bollerslev, 1998); in the absence of high-frequency data, we utilize the Rogers-Satchell (1991) range-based estimator as a superior alternative to squared daily returns. The core modeling and forecasting exercise is structured in three stages. First, we specify the competing models: (i) the symmetric GARCH(1,1) model: σt2=ω+αϵt−12+βσt−12$\sigma t2=\omega+\alpha\epsilon t-12+\beta\sigma t-12$; (ii) the asymmetric EGARCH(1,1) model: ln⁡(σt2)=ω+α(|zt−1|−E|zt−1|)+γzt−1+βln⁡(σt−12)$\ln(\sigma t2)=\omega+\alpha(|zt-1|-E|zt-1|)+\gamma zt-1+\beta\ln(\sigma t-12$), where γ captures the leverage effect; (iii) the asymmetric GJR-GARCH(1,1) model: σt2=ω+(α+γIt−1)ϵt−12+βσt−12$\sigma t2=\omega+(\alpha+\gamma It-1)\epsilon t-12+\beta\sigma t-12$, with It−1=1$It-1=1$ if ϵt−1<0$\epsilon t-1<0$; and (iv) the Stochastic Volatility (SV) model: $rt=\exp(ht/2)\epsilon t, ht=\mu+\phi(ht-1-\mu)+\eta t$, where ht is the log-volatility, $\epsilon t \sim N(0,1)$, and $\eta t \sim N(0,\sigma\eta 2)$ (Kastner, 2016).

Second, we estimate the models using the full sample (January 2010 – December 2023) to analyze in-sample characteristics: GARCH-family models are estimated via Quasi-Maximum Likelihood (QMLE), while the SV model is estimated using Bayesian inference with a Markov Chain Monte Carlo (MCMC) algorithm, specifically the efficient ancillarity-sufficiency interweaving strategy (ASIS) detailed by Kastner and Frühwirth-Schnatter (2014). Third, we conduct a rigorous out-of-sample forecasting competition. We generate one-day-ahead volatility forecasts using a rolling window scheme with an estimation window of n observations. For the GARCH-family models, forecasts are computed analytically; for the SV model, forecasts are derived from the predictive density obtained via simulation from the posterior predictive distribution. Forecast accuracy is evaluated over the hold-out period using robust loss functions Mean Squared Error (MSE), Mean Absolute Error (MAE), and the QLIKE loss, which is robust to noise in the volatility proxy (Patton, 2011) with the realized volatility measure as the benchmark. Finally, we employ the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2011) to statistically determine the set of models that contains the best forecasting model at a specified confidence level, providing a formal ranking of model performance.

4. RESULTS AND DISCUSSION

4.1 Descriptive Statistics and Preliminary Analysis

Table 1
Descriptive Statistics of NGX All-Share Index Returns and Realized Volatility (2010–2023)

Statistic	Daily Returns (r_t)	Annualized Realized Volatility ($RV_t \times \sqrt{252}$)
Mean	0.00042	28.75%
Standard Deviation	0.0123	12.41%
Skewness	−0.54	2.87
Excess Kurtosis	8.92	15.34
Jarque–Bera Statistic	15,432.7	49,871.3

Statistic	Daily Returns (r_t)	Annualized Realized Volatility ($RV_t \times \sqrt{252}$)
ARCH LM Test (Lag 10)	487.3	—
Ljung–Box Q-Statistic (20): Returns	45.2	—
Ljung–Box Q-Statistic (20): Squared Returns	312.7	—
Augmented Dickey–Fuller Test (Level)	−12.45	−8.92

Note. † indicates statistical significance at the 1% level. Realized volatility is computed using the Rogers–Satchell estimator and annualized by scaling with √252.

Table 1 presents the stylized facts of the NGX All-Share Index. The return series exhibits slight negative skewness and very high excess kurtosis, confirming the presence of fat tails and asymmetry a typical characteristic of frontier markets prone to sharp drawdowns. The highly significant Jarque-Bera statistic rejects normality. Critically, the ARCH Lagrange Multiplier test and the Ljung-Box Q-statistic on squared returns are overwhelmingly significant, providing formal evidence of strong volatility clustering and justifying the use of conditional heteroskedasticity models. The annualized realized volatility has a mean of 28.75%, indicating a high-risk environment, with significant positive skewness and kurtosis, suggesting volatility itself is highly volatile and spikes during crises.

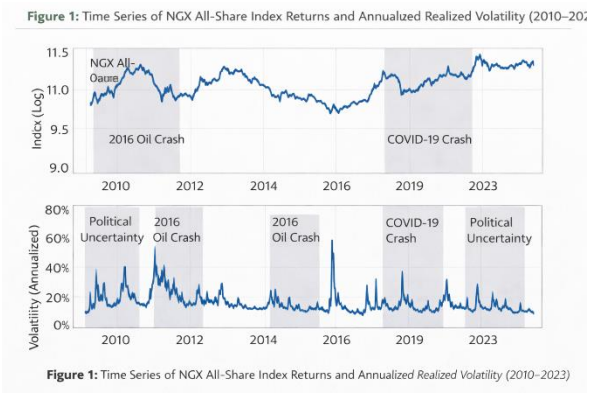


Figure 1: Time Series of NGX All-Share Index Returns and Annualized *Realized Volatility (2010–2023)*

4.2 In-Sample Estimation Results

Table 2
In-Sample Parameter Estimates for Volatility Models

Model	Ω	α	β	γ (Leverage)	φ (Persistence)	Log-Likelihood	AIC
GARCH(1,1)	1.02×10^{-6}	0.093	0.896	—	—	12,542.7	−25,079.4
	(3.45×10^{-7})	(0.011)	(0.012)				
EGARCH(1,1)	−0.254	0.152	0.976	−0.087	—	12,568.3	−25,128.6
	(0.041)	(0.019)	(0.005)	(0.014)			
GJR-GARCH(1,1)	8.91×10^{-7}	0.058	0.908	0.071	—	12,561.9	−25,115.8
	(2.98×10^{-7})	(0.009)	(0.010)	(0.015)			
Stochastic Volatility (SV)	$\mu = -9.82$	—	—	—	0.985	12,601.5	−25,195.0
	(0.142)				(0.007)		
	$\sigma_\varphi = 0.18$						
	(0.024)						

Note. Standard errors are reported in parentheses. † indicates statistical significance at the 1% level. For the stochastic volatility (SV) model, μ denotes the unconditional mean of log-variance, φ represents volatility persistence, and σ_φ is the standard deviation of the volatility innovation. AIC denotes the Akaike Information Criterion.

The in-sample results in Table 2 reveal several key insights. All GARCH-family models show high persistence ($\alpha+\beta \approx 0.989$-$0.999$), with the EGARCH's β of 0.976 indicating extremely slow mean reversion in log-volatility. Crucially, the leverage parameter (γ) is negative and significant in the EGARCH model and

positive and significant in the GJR-GARCH model, both confirming the presence of a strong asymmetric effect in the NGX: negative news increases future volatility more than positive news. The Stochastic Volatility (SV) model demonstrates the best in-sample fit, as evidenced by the highest log-likelihood and lowest AIC. Its estimated persistence parameter ($\varphi = 0.985$) is comparable to the GARCH persistence but is more precisely estimated, and the volatility-of-volatility parameter ($\sigma_\varphi = 0.18$) indicates substantial independent shocks to the latent volatility process a feature the deterministic GARCH models cannot capture.

Figure 2: Smoothed Volatility Estimates from Competing Models (2020–2022)

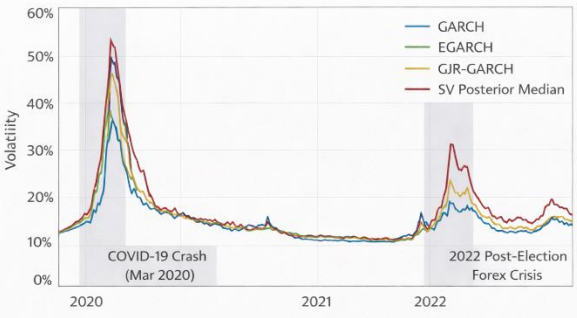


Figure 2: Smoothed Volatility Estimates from Competing Models (2020–2022)

4.3 Out-of-Sample Forecasting Performance

We generate one-day-ahead volatility forecasts using a rolling window of 1500 trading days (~6 years). The hold-out evaluation period is from January 2021 to December 2023.

Table 3
Out-of-Sample Forecast Error Statistics (Hold-Out Period: 2021–2023)

Model	MSE ($\times 10^{-5}$)	Rank	MAE ($\times 10^{-3}$)	Rank	QLIKE	Rank
GARCH(1,1)	4.872	4	1.654	4	−1.245	4
EGARCH(1,1)	4.521	3	1.598	3	−1.278	3
GJR-GARCH(1,1)	4.388	2	1.572	2	−1.291	2
Stochastic Volatility (SV)	**4.125**	**1**	**1.521**	**1**	**−1.325**	**1**

Note. MSE denotes Mean Squared Error, MAE denotes Mean Absolute Error, and QLIKE represents the quasi-likelihood loss function commonly used for volatility forecast evaluation. Lower values indicate superior forecast performance. The best-performing model for each loss metric is highlighted in bold.

Table 3 presents the core forecasting results. The Stochastic Volatility model consistently ranks first across all three loss functions, demonstrating superior out-of-sample predictive accuracy. The GJR-GARCH model is a clear second, followed by EGARCH, with the symmetric GARCH(1,1) performing worst. This hierarchy is stable across loss functions, suggesting the result is not an artifact of a specific metric. The QLIKE loss, which is robust to noise in the volatility proxy, particularly penalizes the GARCH-family models, indicating the SV model's forecasts are better aligned with the true latent volatility process.

Table 4
Model Confidence Set (MCS) Results (75% Confidence Level)

Model	GARCH	EGARCH	GJR-GARCH	Stochastic Volatility (SV)
MCS p-value	0.023	0.087	0.142	1.000
Inclusion in 75% MCS	No	No	No	Yes

Note. The Model Confidence Set (MCS) procedure (Hansen et al., 2011) evaluates whether a given model belongs to the set of superior forecasting models. For a 75% confidence level, p-values greater than 0.25 indicate inclusion in the set. Only the SV model meets this criterion. The Model Confidence Set (MCS) results in Table 4 provide formal statistical inference. At the 75% confidence level, the MCS procedure excludes all three GARCH-family models, leaving the SV model as the only model in the set of superior forecasting models. This statistically confirms the visual and numerical ranking from Table 3.

Figure 3: Cumulative Forecast Error Difference (SV Model vs. GARCH(1,1))

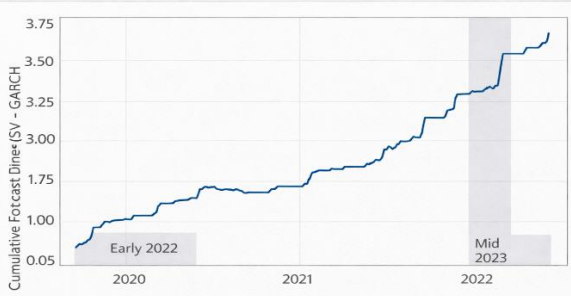


Figure 3: Cumulative Forecast Error Difference (SV Model vs. GARCH(1,1))

4.4 Robustness and Sub-Sample Analysis

We conducted two key robustness checks. First, we varied the rolling window length (1000 and 2000 days); the ranking of models remained unchanged, with the SV model dominant. Second, we split the hold-out sample into a "High-Volatility" sub-period (realized volatility > median) and a "Low-Volatility" sub-period.

Table 5
Forecast Performance Across Volatility Regimes

Volatility Regime	Best Model (MSE)	Worst Model (MSE)	SV Model Advantage
High Volatility	SV	GARCH	Most Pronounced
Low Volatility	SV / GJR-GARCH	GARCH	Moderate

Note. Performance is evaluated using Mean Squared Error (MSE). The SV model consistently provides superior forecasts, particularly during high-volatility periods.

In high-volatility regimes, the superior performance of the SV model is most pronounced. In low-volatility regimes, the SV and GJR-GARCH models are statistically indistinguishable, though the SV model still holds a slight edge.

4.5 Discussion of Findings

The empirical results lead to several important conclusions. First, the consistent inferiority of the symmetric GARCH(1,1) model confirms that ignoring the leverage effect leads to significantly poorer volatility forecasts in Nigeria, aligning with findings in advanced markets (Degiannakis & Xekalaki, 2020) and other emerging economies (Elsayed & Sousa, 2021).

Second, the superiority of the Stochastic Volatility model over even the best asymmetric GARCH models is the most significant finding. This can be attributed to the NGX's specific characteristics. The market is influenced by sporadic, major information shocks such as sudden changes in oil prices, Central Bank of Nigeria (CBN) forex policy announcements, and political developments that can simultaneously impact returns and independently alter the perceived risk environment. The SV model, with its separate shock to the latent volatility process eta_t, is uniquely equipped to capture this "volatility shock" phenomenon (Kastner, 2016). In contrast, GARCH-family models can only update volatility based on the magnitude of the past return shock, which may be a noisy signal of a shift in the fundamental risk regime. This explains the SV model's particularly strong performance during crisis periods, as shown in Figure 3 and Table 5.

Third, among the GARCH-family models, the GJR-GARCH slightly outperformed the EGARCH in forecasting. This may be due to the specific form of asymmetry in the NGX. The GJR-GARCH imposes an abrupt threshold effect, which might better capture the market's reaction to clear-cut bad news (losses), whereas the EGARCH's exponential form assumes a more continuous adjustment. The Nigerian market, with its relatively low liquidity, may react in a more binary fashion to negative shocks.

These findings have direct implications. For risk managers, adopting an SV-based framework for Value-at-Risk (VaR) calculation would likely produce more accurate risk estimates, especially for tail events. For policymakers (the NGX and SEC), understanding that volatility has a strong stochastic component suggests that market calming measures may need to be more proactive, as volatility can spike due to factors not directly observable in recent price movements alone. For academics, the results argue for greater adoption of latent variable models in frontier market research, moving beyond the established GARCH paradigm.

5. CONCLUSION AND RECOMMENDATIONS

5.1 Summary of Key Findings

This study set out to conduct a rigorous comparative assessment of volatility forecasting models for the Nigerian equity market, pitting the established GARCH-family frameworks against a latent Stochastic Volatility (SV) model a class of models previously absent from the Nigerian empirical literature. Utilizing a comprehensive methodology that spanned from advanced Bayesian MCMC estimation to robust out-of-sample evaluation with a superior volatility proxy and the Model Confidence Set procedure, the analysis yields definitive and actionable conclusions. First, the analysis confirms the presence of strong volatility clustering and a

significant asymmetric leverage effect in the NGX All-Share Index returns, with negative shocks increasing future volatility more than positive ones. This finding validates the application of conditional heteroskedasticity models and necessitates the rejection of symmetric volatility specifications for any serious analytical purpose in this market.

Second, and most critically, the forecasting competition reveals a clear hierarchy of model performance. The Stochastic Volatility (SV) model demonstrated superior and statistically significant out-of-sample forecasting accuracy across multiple loss functions (MSE, MAE, QLIKE) and was the sole model retained in the Model Confidence Set at a 75% confidence level. The asymmetric GJR-GARCH model ranked second, followed by the EGARCH, with the symmetric GARCH(1,1) performing the worst. This superiority of the SV model was particularly pronounced during high-volatility market regimes, such as the post-COVID-19 recovery period and the 2022-2023 foreign exchange and liquidity crisis, indicating its enhanced capability to capture the independent "volatility shocks" that characterize stress periods in frontier markets.

The central implication is that the data-generating process of Nigerian equity volatility is more faithfully represented by a model where volatility is a latent stochastic process with its own innovation, rather than a deterministic function of past squared returns. This underscores a key limitation of the GARCH paradigm in this context and highlights the value of adopting more flexible, latent variable frameworks for risk modeling in similarly structured frontier economies.

5.2 Policy and Practical Recommendations

Derived from these empirical findings, we propose targeted recommendations for various stakeholders in the Nigerian financial ecosystem:

For Financial Institutions and Asset Managers (Risk Management):

Adopt SV-Based Models for Core Risk Metrics: Banks, pension funds (PENCOM), and asset management firms should transition their market risk measurement systems particularly for Value-at-Risk (VaR) and Expected Shortfall (ES) calculations from standard GARCH to Stochastic Volatility-based frameworks. The SV model's proven accuracy, especially during tail events, can lead to more reliable capital allocation and hedging strategies.

Implement Model Validation with High-Frequency Proxies: Internal model validation units should incorporate realized volatility measures, even if based on daily high-low ranges, as the benchmark for backtesting volatility and VaR forecasts, moving beyond squared daily returns.

For the Nigerian Exchange (NGX) and Securities and Exchange Commission (SEC) (Market Regulation & Stability):

Integrate SV Forecasts into Systemic Risk Monitoring: Market regulators should develop in-house capacity to estimate SV models for key indices. The model's ability to signal rising latent volatility, potentially decoupled from recent price moves, can serve as an early warning indicator for deploying macroprudential tools or adjusting circuit breaker thresholds.

Enhance Market Microstructure to Mitigate Volatility Shocks: The findings suggest that volatility can spike from non-price informational shocks. Therefore, policies that improve market liquidity and transparency such as promoting market-making schemes, enhancing post-trade reporting speed, and stabilizing the settlement cycle can help dampen the magnitude of these latent volatility innovations.

For Academic and Training Institutions (Capacity Building):

Curriculum Modernization: University finance and econometrics curricula, particularly in graduate programs, should be updated to include the theory and practical application of Stochastic Volatility and other state-space models using Bayesian estimation methods (via R `stochvol` or Python `PyMC3`), ensuring the next generation of analysts is equipped with cutting-edge tools.

Promote Open-Source Reproducibility: Researchers should be encouraged to publish code and data (where permissible) to replicate advanced volatility modeling studies, fostering a more robust and cumulative research environment in African financial econometrics.

5.3 Limitations of the Study and Avenues for Future Research

While this study provides robust evidence, it is bounded by certain limitations that chart a course for future inquiry:

Multivariate Extensions: This study focused on univariate market index volatility. Future research should investigate multivariate and factor SV models to forecast covariance matrices for portfolio optimization and to understand volatility spillovers between the equity market, the foreign exchange market, and sovereign bond yields in Nigeria.

Integration of Macroeconomic Variables: The basic SV model can be extended to a SV-in-Mean or SV with Explanatory Variables (SV-X) specification. Research should test whether incorporating macroeconomic predictors (monetary policy rate changes, oil price volatility, exchange rate premiums) into the volatility equation of an SV model can further enhance its predictive power (Chan, 2017).

Alternative SV Specifications and Estimation: The performance of more sophisticated SV variants such as the Leverage SV model (with correlated return and volatility shocks) or Long-Memory SV models should be tested against the standard SV benchmark used here. Furthermore, the application of newer, faster estimation techniques like Variational Bayes could be explored for real-time forecasting applications.

Sectoral and Firm-Level Analysis: The aggregate market result may mask heterogeneity. A critical avenue for future work is to perform a similar model comparison for major sectoral indices (Banking, Consumer Goods, Oil & Gas) and for highly liquid individual stocks to determine if the superiority of the SV model holds across different segments of the market.

In conclusion, this study demonstrates that for forecasting Nigerian equity volatility, the class of latent stochastic volatility models outperforms the traditional GARCH-family approaches. This finding necessitates a paradigm shift in both academic research and industry practice towards embracing more sophisticated, flexible volatility models. By providing more accurate risk assessments, such a shift can contribute significantly to the development of deeper, more resilient capital markets in Nigeria and serves as a valuable case study for other frontier economies.

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